

Gauge coupling unification in GUTs through gravitational effects

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with Stephen Hsu and Xavier Calmet:

- Phys. Rev. D **81**, 035007 (2010) [arXiv:0911.0415 [hep-ph]]
“Grand unification through gravitational effects”
- Phys. Rev. Lett. **101**, 171802 (2008) [arXiv:0805.0145 [hep-ph]]
“Grand unification and enhanced quantum gravitational effects”

- ① Introduction:
Grand Unified Theories — Non-SUSY & SUSY
- ② Theoretical Framework:
Gravitational Dimension-5 Interactions
- ③ Unification Results in Concrete Models:
Non-Supersymmetric Unification Through Dim-5 Operators
- ④ Case Study for SUSY-GUTs:
Uncertainty in Gauge Coupling Unification Predictions
- ⑤ Conclusions

Motivation for Grand Unification

Standard Model of Particle Physics:

- 3 gauge interactions $SU(3)_C \times SU(2)_L \times U(1)_Y$:
 $\alpha_3(m_Z) = 0.1176$, $\alpha_2(m_Z) = 0.03322$, $\alpha_1(m_Z) = 0.016887$
 - 3 fermion families: $Q \sim (\mathbf{3}, \mathbf{2}, 1/6)$, $u^c \sim (\bar{\mathbf{3}}, \mathbf{1}, -2/3)$,
 $d^c \sim (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$, $L \sim (\mathbf{1}, \mathbf{2}, -1/2)$, $e^c \sim (\mathbf{1}, \mathbf{1}, 1)$
 - quantization of Y -charges ?
 - anomaly cancellation ?
 - ...
-

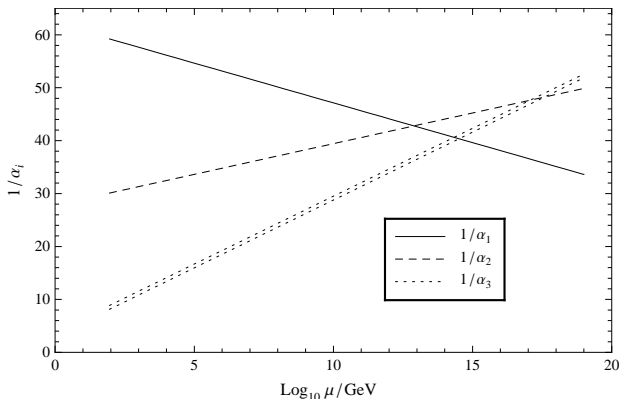
Grand Unification (e.g. Georgi-Glashow $SU(5)$):

- 1 gauge interaction $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$: α_G
- 3 fermion families: $\bar{\mathbf{5}} = [d^c, L]$, $\mathbf{10} = [Q, u^c, e^c]$

→ but $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_G$!

Gauge coupling RG evolution

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(m_Z)} - \frac{b_s}{2\pi} \ln \frac{\mu}{m_Z} \quad (b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7)$$



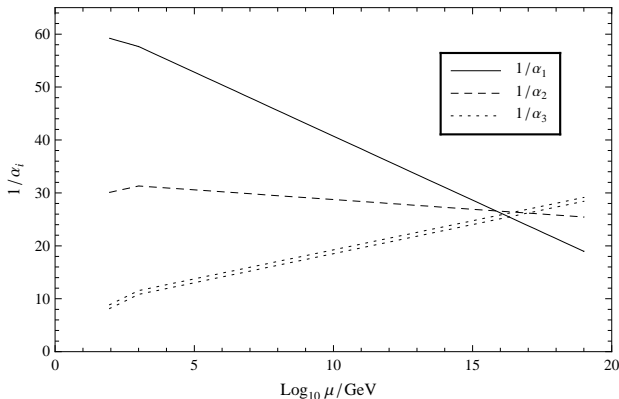
- $\alpha_i(M_X) = \alpha_G$?
- $5 \times 10^{33} \text{ years} < \tau_{\text{proton} \rightarrow e^+ \pi^0} \sim M_X^4 \Rightarrow M_X > 3 \times 10^{15} \text{ GeV} !$

Supersymmetric GUT

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(m_Z)} - \frac{b_s}{2\pi} \ln \frac{\mu}{m_Z}$$

$$b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad (\text{for } \mu \leq m_{SUSY} \sim 1 \text{ TeV})$$

$$b_1 = \frac{33}{5}, \quad b_2 = 1, \quad b_3 = -3 \quad (\text{for } \mu > m_{SUSY})$$



$$\alpha_1(M_X) \approx \alpha_2(M_X) \approx \alpha_3(M_X) = \alpha_G \quad \text{for } M_X = 2 \times 10^{16} \text{ GeV}$$

Problems with the Grand Unification Scenario

- low-energy supersymmetry (SUSY) not yet found
- proton lifetime constraint (M_X too low)
(with SUSY: $M_X = 2 \times 10^{16}$ GeV too low)
- exact unification of gauge couplings
- doublet-triplet splitting (hierarchy problem)
- fermion mass relations violated (for minimal models)
- neutrino masses, family unification, ...
- possible Landau poles in SUSY-GUTs

→ need new physics for GUT scenario:

(a) intermediate scale physics: $M_I < M_X$ [e.g. Lavoura/Wolfenstein 1993]

(b) gravity-related physics: $M_{Pl} = 10^{19}$ GeV

→ note: $M_X/M_{Pl} \sim 10^{-3}$ and gravity \notin GUT

Effective Gravitational Interactions

effective field theory: *any* gauge and Lorentz singlet interaction

lowest-order interaction to affect unification of $\alpha_i(\mu)$:

$$\mathcal{L}_{GUT} = \frac{c}{4M_{Pl}} H G_{\mu\nu} G^{\mu\nu} \quad c \sim \mathcal{O}(1)$$

(Hill 1984, Shafi&Wetterich 1984)

Concrete realizations:

- $\mathcal{N} = 1$ supergravity: lowest-order expansion of non-canonical gauge kinetic function $f^{ab}(H_i)$
(Ellis *et. al* 1985, Drees 1985)
- spontaneous compactification from higher dimensions
(Wetterich 1982, Weinberg 1983)
- in gravitational instanton background
(Perry 1979)

Aside: Choice of Planck Scale M_{Pl}

$$\mathcal{L}_{GUT} = \frac{c}{4M_{Pl}} H G_{\mu\nu} G^{\mu\nu}$$

effective interaction by “integrating out gravity”

$\Rightarrow M_X$ = energy scale of gravity

$$\textcircled{1} \quad [G_N] = \text{mass}^{-2} \quad \Rightarrow \quad M_{Pl} = G_N^{-1/2} = 1.2 \times 10^{19} \text{ GeV}$$

$$\textcircled{2} \quad \mathcal{L}_{\text{grav+matter}} = -\frac{1}{16\pi G_N} g^{\mu\nu} \square g_{\mu\nu} - \frac{1}{2} \phi \square \phi + \dots$$

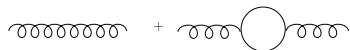
$$\Rightarrow M_{Pl} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18} \text{ GeV}$$

$$\rightarrow \text{parametrize: } M_{Pl} = \frac{1.2 \times 10^{19} \text{ GeV}}{\xi}$$

$$\xi = 1 \quad \text{or} \quad \xi_{\text{red}} = \sqrt{8\pi} \approx 5$$

Aside: Choice of Planck Scale M_{Pl}

Running G_N :

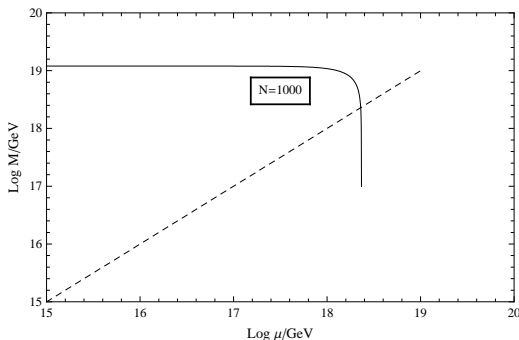


$$\frac{1}{G(\mu)} = \frac{1}{G_N} - \mu^2 \frac{N}{12\pi}$$

with $N \equiv N_0 + N_{1/2} - 4N_1$
 ~ 1000 in GUTs

(Calmet, Hsu, Reeb 2008;

Larsen, Wilczek 1995; ...; ADD, ...)



$$M_X \stackrel{!}{=} G(M_X)^{-1/2} \Rightarrow M_{Pl} = \frac{1.2 \times 10^{19} \text{ GeV}}{\sqrt{1 + N/12\pi}} \equiv \frac{1.2 \times 10^{19} \text{ GeV}}{\xi^{\text{run}}}$$

$$\xi^{\text{run}} \approx 0.7 \dots 8, \quad \xi_{\text{red}}^{\text{run}} = \sqrt{8\pi} \xi^{\text{run}} \approx 5 \xi^{\text{run}}$$

Dimension-5 Operators

$$\mathcal{L}_{GUT} = \frac{c}{4M_{Pl}} H G_{\mu\nu} G^{\mu\nu} = \sum_i \frac{c_i}{4M_{Pl}} H_i^{ab} G_{\mu\nu}^a G^{b\mu\nu}$$

- $G_{\mu\nu}^a$ = gauge field strength of GUT
- a, b = adjoint indices (e.g. $a, b = 1 \dots 24$ for $SU(5)$)
 $a = 1 \dots 8$: $SU(3)_C$, $a = 9 \dots 11$: $SU(2)_L$, $a = 12$: $U(1)_Y$
- H_i = GUT-Higgs fields (acquire vev at M_X)
for $SU(5)$: H_i in **1**, **24**, **75**, or **200** irreducible representation
- “Wilson coefficients” $c_i \sim \mathcal{O}(1)$

Below GUT symmetry breaking: $H_i^{ab} \rightarrow \langle H_i^{ab} \rangle \sim M_X$

$$\mathcal{L} = \sum_i \frac{c_i \langle H_i^{ab} \rangle}{M_{Pl}} \frac{1}{4} G_{\mu\nu}^a G^{b\mu\nu} - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

Dimension-5 Operators ($SU(5)$ case)

$\langle H_i^{ab} \rangle$ invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$!

$$\Rightarrow \langle H_i^{ab} \rangle = v_i \cdot \begin{cases} \delta_3^i, & a = b \in \{1, \dots, 8\} & \rightarrow SU(3)_C \\ \delta_2^i, & a = b \in \{9, \dots, 11\} & \rightarrow SU(2)_L \\ \delta_1^i, & a = b = 12 & \rightarrow U(1)_Y \\ \delta_s^i, & a, b \geq 13 \ (s \geq 4) \end{cases}$$

$SU(5)$ irrep r	$\delta_1^{(r)}$	$\delta_2^{(r)}$	$\delta_3^{(r)}$
1	$-1/\sqrt{24}$	$-1/\sqrt{24}$	$-1/\sqrt{24}$
24	$1/\sqrt{63}$	$3/\sqrt{63}$	$-2/\sqrt{63}$
75	$5/\sqrt{72}$	$-3/\sqrt{72}$	$-1/\sqrt{72}$
200	$-10/\sqrt{168}$	$-2/\sqrt{168}$	$-1/\sqrt{168}$

$$\epsilon_s := \sum_i \frac{c_i}{M_{Pl}} v_i \delta_s^{(i)}$$

$$(\epsilon_1 \neq \epsilon_2 \neq \epsilon_3)$$

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \sum_i \frac{c_i}{4M_{Pl}} \langle H_i^{ab} \rangle G_{\mu\nu}^a G^{b\mu\nu} \\ &= -\frac{1}{4} (1 + \epsilon_3) F_{\mu\nu}^a F^{a\mu\nu}_{SU(3)} - \frac{1}{4} (1 + \epsilon_2) F_{\mu\nu}^a F^{a\mu\nu}_{SU(2)} - \frac{1}{4} (1 + \epsilon_1) F_{\mu\nu} F^{\mu\nu}_{U(1)} + \dots \end{aligned}$$

Modification of the Unification Condition

At scales $\mu < M_X$:

$$\mathcal{L} = -\frac{1}{4}(1+\epsilon_3)F_{\mu\nu}^a F_{SU(3)}^{a\mu\nu} - \frac{1}{4}(1+\epsilon_2)F_{\mu\nu}^a F_{SU(2)}^{a\mu\nu} - \frac{1}{4}(1+\epsilon_1)F_{\mu\nu} F_{U(1)}^{\mu\nu}$$

canonical normalization:	$\mu > M_X$	$\mu < M_X$
	$F_{(s)}^{\mu\nu}$	$\rightarrow (1+\epsilon_s)^{1/2} F_{(s)}^{\mu\nu}$
	$A_{(s)}^\mu$	$\rightarrow (1+\epsilon_s)^{1/2} A_{(s)}^\mu$
	g_s	$\rightarrow (1+\epsilon_s)^{-1/2} g_s$
	α_s	$\rightarrow (1+\epsilon_s)^{-1} \alpha_s$

\Rightarrow Correct Gauge Coupling Unification Condition:

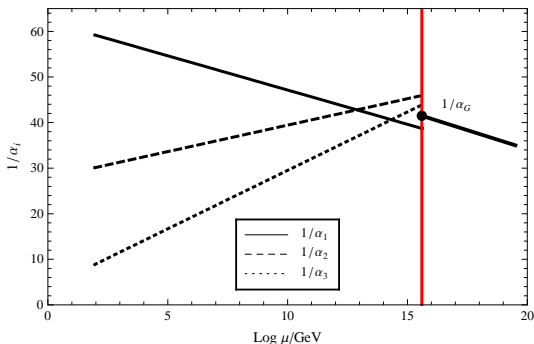
$$(1+\epsilon_1)\alpha_1(\mu = M_X) = (1+\epsilon_2)\alpha_2(\mu = M_X) = (1+\epsilon_3)\alpha_3(\mu = M_X)$$

$$\left[\epsilon_s = \sum_i \frac{c_i}{M_{Pl}} v_i \delta_s^{(i)}, \quad g_G \sqrt{\sum_i \frac{C_2(r_i)}{12} v_i^2} = M_X \right]$$

- or: change β -functions
- NO approximation!

Modified Unification Condition

$$\begin{aligned}
 & (1 + \epsilon_1) \alpha_1(\mu = M_X) \\
 = & (1 + \epsilon_2) \alpha_2(\mu = M_X) \\
 = & (1 + \epsilon_3) \alpha_3(\mu = M_X) \\
 \equiv & \alpha_G
 \end{aligned}$$



Example: $M_X = 4 \times 10^{15}$ GeV. Suppose $1/\alpha_G(M_X) = 41.5$ and

$$\epsilon_1 = \sum_i \frac{c_i}{M_{Pl}} v_i \delta_1^{(i)} = -0.067, \quad \epsilon_2 = 0.106, \quad \epsilon_3 = 0.058.$$

Then:

$$1/\alpha_1(M_X) = 38.7, \quad 1/\alpha_2(M_X) = 45.9, \quad 1/\alpha_3(M_X) = 43.9.$$

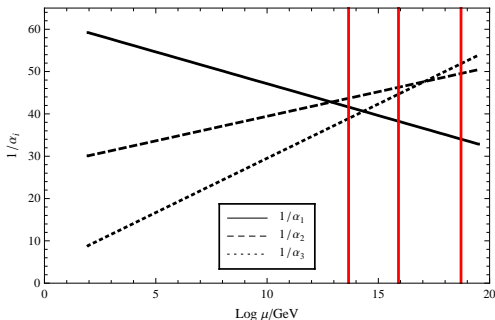
→ lead to the *actually observed* $\alpha_s(m_Z)$ at $\mu = m_Z$ (without SUSY)

- first comprehensive study for **multiple** dim-5 operators in GUT
→ **qualitatively** new possibilities (**next slides**)
- concentrate on *non*-supersymmetric case (**next slides**)
- *absolute* normalization for δ_s^i across different irreps r_i
- both $SU(5)$ and $SO(10)$ (“normal” & “flipped” embedding)
- compute *all* SM singlets δ_s^i for $SO(10)$ (in 2 distinct bases)
(this makes the *most minimal* $SO(10)$ models feasible)
- compute δ_s^i for $s \neq SU(3)_C, SU(2)_L, U(1)_Y$
- (also: non-universal gaugino masses from $\mathcal{N} = 1$ SUGRA)
[Ellis *et al.* 1985, Drees 1985, ... many more]

Non-SUSY Unification Results: 1 dim-5 operator

2 parameters (c and v), 2 equations ($\alpha_1 = \alpha_2 = \alpha_3$) \Rightarrow 1 solution

H irrep	M_X	c	v	$\max_s \epsilon_s $
1	impossible			
24	4.6×10^{13} GeV	$18700/\xi$	1.3×10^{14} GeV	0.076
75	8.1×10^{15} GeV	$-129/\xi$	1.8×10^{16} GeV	0.116
200	5.2×10^{18} GeV	$0.53/\xi$	1.1×10^{19} GeV	0.363



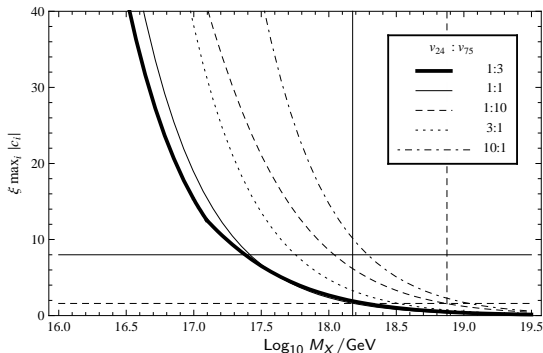
e.g. for a **75** Higgs:

$$\begin{aligned}
 \epsilon_1 &= c \frac{v}{M_{Pl}} \delta_1^{75} \\
 &= \frac{-129}{\xi} \frac{1.8 \times 10^{16} \text{ GeV}}{1.2 \times 10^{19} \text{ GeV}/\xi} \frac{5}{\sqrt{72}} \\
 &= -0.116 \\
 \epsilon_2 &= 0.070 \\
 \epsilon_3 &= 0.023
 \end{aligned}$$

(cf., e.g., Hill 1984, Shafi&Wetterich 1984, Chakraborty&Raychaudhuri 2009)

Non-SUSY Unification Results: 2 dim-5 operators

4 parameters (c_i and v_i), 2 eqns \Rightarrow 2-dim solution set (continuous!)



- non-SUSY
- exact unification
- large enough M_X :
continuously variable
with model parameters
- natural $c \sim \mathcal{O}(1)$

Examples:

- ① $\xi_{\text{red}} = 5$, $|c_{24}|, |c_{75}| < 1$, $v_{24} : v_{75} = 1 : 3 \rightarrow$ any $M_X > 5 \times 10^{17}$ GeV
- ② $\xi_{\text{red}}^{\text{run}} = 8$, $|c_{24}|, |c_{75}| < 5$, $v_{24} : v_{75} = 1 : 3 \rightarrow$ any $M_X > 3 \times 10^{16}$ GeV

Non-SUSY Unification Results: General Estimate

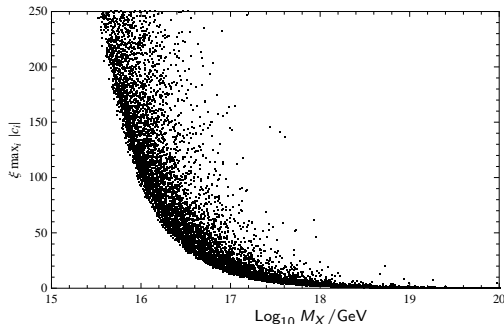
- couplings $\alpha_s(\mu)$ differ by $\leq 50\%$ (for $10^{13} \text{ GeV} \leq \mu \leq 10^{19} \text{ GeV}$)
- $(1 + \epsilon_1)\alpha_1(\mu) = (1 + \epsilon_2)\alpha_2(\mu) = (1 + \epsilon_3)\alpha_3(\mu)$ at $\mu = M_X$
- \rightarrow need $\epsilon_s \sim \pm 10\%$
- $\epsilon_s \sim c \frac{\langle H \rangle}{M_{Pl}} \sim c \frac{\delta_s^{(r)} M_X}{g_G M_{Pl}}$
- $g_G \approx 0.5$; $\delta_s^{(r)} \lesssim 0.5$, and linearly independent across irreps r

for h Higgs multiplets:

$$\Rightarrow \max_i |c_i| \gtrsim \frac{0.1 M_{Pl}}{\sqrt{h} M_X}$$

(for $SU(5)$; figure: $h = 3$,
model with **24, 75, 200**)

\rightarrow large M_X self-consistent



Proton Lifetime Limit

$$\tau_{(\text{proton} \rightarrow e^+ \pi^0)} > 5 \times 10^{33} \text{ years} \quad (\text{expected in 10 years: } \tau > 10^{35} \text{ years})$$

\Rightarrow in non-SUSY GUTs:

$$M_X > 4 \times 10^{15} \text{ GeV} \quad (\text{in a decade: } M_X > 8 \times 10^{15} \text{ GeV})$$

This is OK by previous estimate, for natural $c_i \sim O(1)$:

$$M_X > \frac{0.1}{\sqrt{h} \max_i |c_i|} \frac{1.2 \times 10^{19} \text{ GeV}}{\xi} \gg 4 \times 10^{15} \text{ GeV}$$

(\rightarrow also OK in all examples shown previously)

[observation of proton decay \Rightarrow strong limits on exact non-SUSY unification, e.g. $SU(5)$ w/ **24&75**, $\xi = \xi_{\text{red}}^{\text{run}} = 8$, $|c_i|_{\text{max}} = 15 \Rightarrow M_X = 8 \times 10^{15} \text{ GeV}$]

Proton Decay Experiments

- parameter space for viable Grand Unification models is large (for both non-SUSY and also SUSY models)
- in particular, any $M_X \sim 10^{17-19}$ GeV possible in a natural way (with $c \sim \mathcal{O}(1)$)
- 16-fold detector volume $\Rightarrow \tau_{\text{decay}} \rightarrow 16\tau_{\text{decay}}$
 $\Rightarrow M_{X, \text{lower}} \rightarrow 2M_{X, \text{lower}}$
- lower bound today: $M_X \geq 4 \times 10^{15}$ GeV

\Rightarrow huge effort to constrain GUT parameter space via proton decay experiments

Curious observation

Unification of gauge and gravitational interactions ?

(Agashe/Delgado/Sundrum 2003: in Randall-Sundrum 1; Lykken/Willenbrock 1994: with technicolor; Howl/King 2007: intermediate gauge symmetries)

Achieving $M_X = M_{Pl}$ requires only **small** Wilson coefficients c :

$$\max_i |c_i| \approx \frac{O(0.1)}{\sqrt{h}} \frac{M_{Pl}}{M_X} \stackrel{M_X=M_{Pl}}{\approx} 0.2$$

Examples ($M_{Pl} = 2.4 \times 10^{18}$ GeV):

- ① $SU(5)$ with **200** Higgs, $c = 0.11 \Rightarrow M_X = 5.2 \times 10^{18}$ GeV
- ② $SU(5)$ with **24** and **75**, $\max_i |c_i| = 0.22 \Rightarrow M_X = 2.4 \times 10^{18}$ GeV

But: other important gravitational operators at $\mu \sim M_X \approx M_{Pl}$:

$$\mathcal{L} = \frac{c_6}{4M_{Pl}^2} H_1 H_2 G_{\mu\nu} G^{\mu\nu} + \frac{c_7}{4M_{Pl}^3} H_1 H_2 H_3 G_{\mu\nu} G^{\mu\nu} + \dots$$

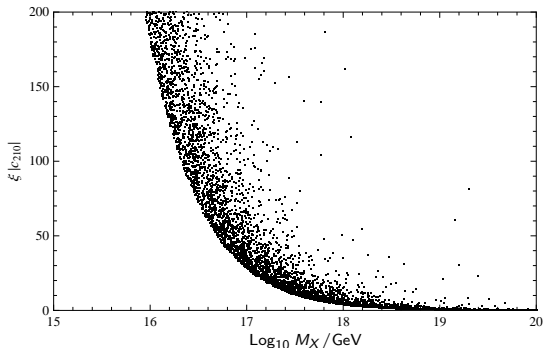
Non-supersymmetric $SO(10)$

$\langle H^{ab} \rangle$ singlet under $SU(3) \times SU(2) \times U(1)$:

→ does *NOT* fix vev direction !

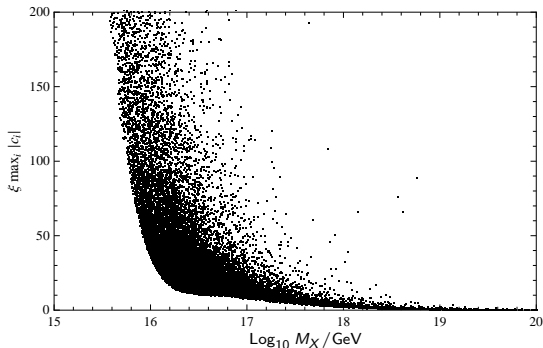
⇒ $SO(10)$ with **one 210** Higgs multiplet has:

$$\epsilon_s = \frac{c}{M_{Pl}} \sum_{j=1}^3 v_j \delta_s^{(210)j} \quad \begin{array}{l} \rightarrow \text{continuously variable } \epsilon_1 : \epsilon_2 : \epsilon_3 \\ \rightarrow \text{continuously variable } M_X \end{array}$$



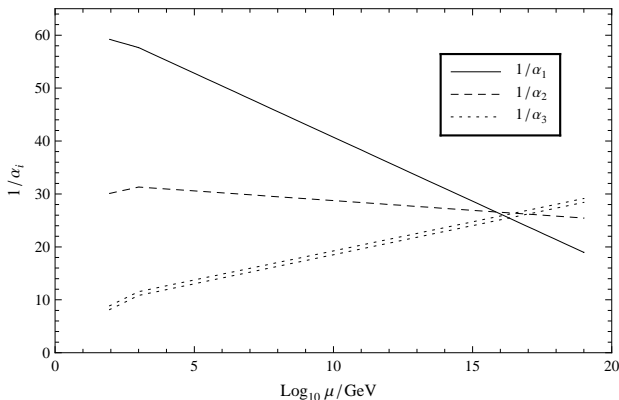
Unification Results: Supersymmetric $SU(5)$

- good gauge coupling unification already w/o dimension-5 operators: $M_X \sim 2 \times 10^{16}$ GeV (for $m_{SUSY} = 1$ TeV)
- but conflict with proton lifetime constraint



- can shift unification scale M_X up
- satisfy proton lifetime constraint
- “prefer” $M_X \approx 2 \times 10^{16}$ GeV (cf. non-SUSY case)

Case Study: Uncertainty in Unification (for SUSY-GUTs)



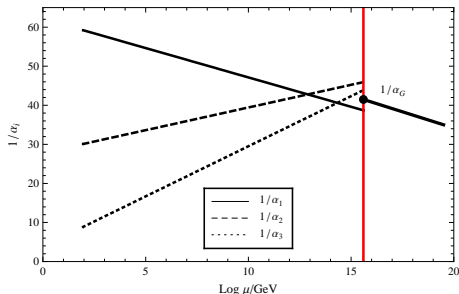
- assuming “particle desert” between m_{SUSY} and M_X
- measurement uncertainty in $\alpha_i(m_Z)$: less than $\pm 4\%$
- 2-loop RG evolution equations

→ additional uncertainty from ignorance about size of $cHG_{\mu\nu}G^{\mu\nu}$

Case Study: Required Splitting vs. 2-loop Corrections

$$\begin{aligned} & (1 + \epsilon_1)\alpha_1(M_X) \\ &= (1 + \epsilon_2)\alpha_2(M_X) \\ &= (1 + \epsilon_3)\alpha_3(M_X) \end{aligned}$$

→ *requires* numerical splitting
between $\alpha_i(M_X)$ for unification



e.g. for $SU(5)$ with one **24** Higgs and $M_{Pl} = 2.4 \times 10^{18}$ GeV:

$$\begin{aligned} \frac{\alpha_3(M_X) - \alpha_2(M_X)}{\alpha_3(M_X)} &\approx \epsilon_2(c) - \epsilon_3(c) \approx +1.5\% \quad \text{if } c = +1 \\ &\approx -1.5\% \quad \text{if } c = -1 \end{aligned}$$

But: 2-loop correction to $\alpha_i(M_X)$ is $< 3.5\%$.

Case Study: Uncertainties from Gravity

Size of (and uncertainties in) effects from gravity comparable to higher-loop contributions

→ 2-loop RG does not improve evidence for grand unification

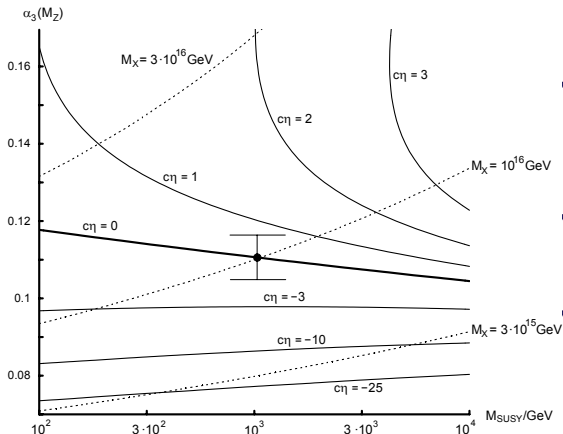
Uncertainty in low-energy measurements:

- $\alpha_1(M_Z) = 0.016887 \pm 0.000040$ ($\pm 0.2\%$)
- $\alpha_2(M_Z) = 0.03322 \pm 0.00025$ ($\pm 0.8\%$)
- $\alpha_3(M_Z) = 0.118 \pm 0.005$ ($\pm 4\%$)
- $M_{\text{SUSY}} = 10^{3\pm 1} \text{ GeV}$ (SUSY breaking scale)

→ is *smaller* than uncertainties from gravitational dim-5 operators:

Case Study: Uncertainty in c vs. low-energy measurements

Small variations in $c \equiv$ large changes in $\alpha_i(M_Z)$, M_{SUSY} :



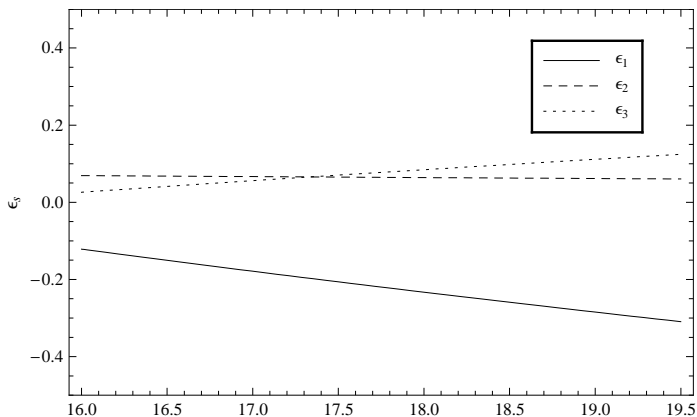
- allowing $c\eta \sim O(5)$:
can make unification happen
- uncertainties in $c\eta$ greater
than in measurements
- if $c > 4/\eta$:
unification incompatible
with low-energy inputs

→ "precise" measurements not good evidence for grand unification

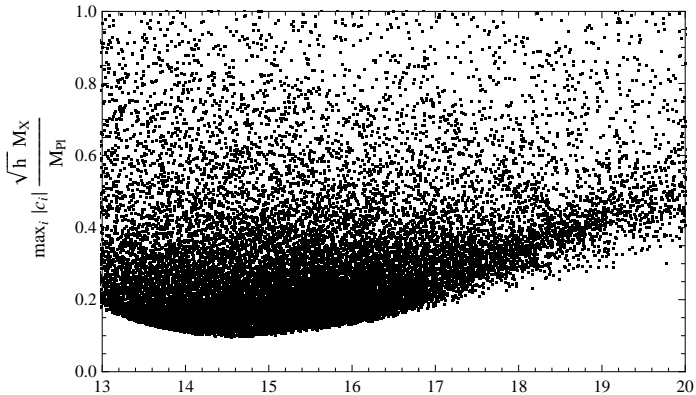
Conclusions

- effective gravitational interactions influence GUT models
(note: $M_{\text{GUT}} \sim M_{\text{Pl}}$, and gravity \notin GUT)
 - fairly minimal unification models possible:
 - non-supersymmetric
 - small unification groups $SU(5)$, $SO(10)$
 - 2 Higgs multiplets (or one **210** of $SO(10)$)
 - escape proton decay limit easily ($10^{16} \text{ GeV} \leq M_X \leq 10^{19} \text{ GeV}$)
in non-SUSY and in SUSY models
 - gauge-gravity unification ?
 - uncertainty in prediction of grand unification from low energy
observations due to high-energy effects
-
- supersymmetry ?
 - proton decay experiments ?

Backup: ϵ_s for **24** and **75** model



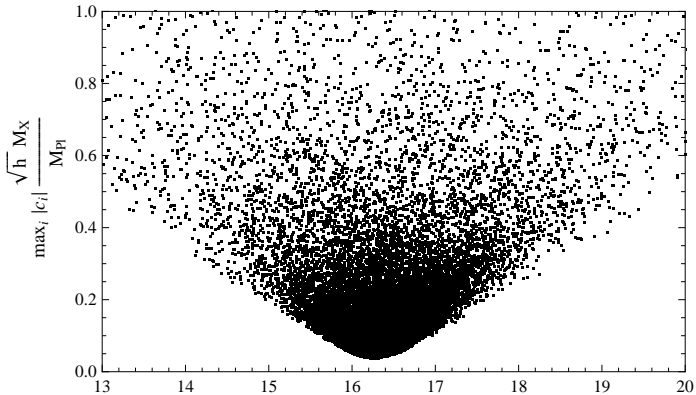
Backup: $O(1)$ const in non-SUSY $SU(5)$ with 24, 75, 200



$$\max_i |c_i| \gtrsim \frac{O(0.1)}{\sqrt{h}} \frac{M_{Pl}}{M_X}$$

and: $O(0.1) = 0.3 \Rightarrow$ feasible

Backup: $O(1)$ const in SUSY $SU(5)$ with 24, 75, 200



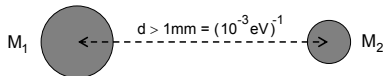
$$\max_i |c_i| \gtrsim \frac{O(0.1)}{\sqrt{h}} \frac{M_{Pl}}{M_X} \quad \text{and: } O(0.1) = 0.5 \Rightarrow \text{feasible}$$

Newton's constant in the infrared

G_N only measured at large distances, i.e. at very low energies:

$$G_N = (10^{19} \text{ GeV})^{-2} = G_N(\mu \approx 0 \text{ GeV})$$

$$M_{\text{Pl}} \equiv G_N^{-1/2} = 10^{19} \text{ GeV} = M_{\text{Pl}}(\mu \approx 0 \text{ GeV})$$



Conventional wisdom: effects from gravity are weak at our low energies $\ll 10^{19} \text{ GeV}$; suppressed by huge M_{Pl} .

But: How is $G_N(\mu = 0)$ related to physics at short distances (quantum gravity)?

Renormalization of G_N : cutoff regularization

$$S_{\text{m+grav}} = \int d^4x \sqrt{|\det g^{\mu\nu}|} \left(\frac{1}{16\pi G_b} R(g^{\mu\nu}) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \dots + \psi + A + \dots \right)$$

gives loop-corrections to graviton propagator:

$$\begin{aligned} \text{diagram} + \text{diagram} &= \frac{iG_b}{q^2} + \frac{iG_b}{q^2} (i\Sigma) \frac{iG_b}{q^2} + \dots \\ \text{with } \Sigma &= \frac{c}{16\pi^2} q^2 \Lambda^2 + \dots \end{aligned}$$

→ Absorb loop corrections into redefinition $G_b \rightarrow G_{\text{ren}}$:

$$\frac{iG_{\text{ren}}}{q^2} = \frac{iG_b}{q^2} + \frac{iG_b}{q^2} \left(\frac{ic}{16\pi^2} q^2 \Lambda^2 \right) \frac{iG_b}{q^2} + \dots$$

$$\Rightarrow \quad \frac{1}{G_{\text{ren}}} = \frac{1}{G_b} + \frac{c}{16\pi^2} \Lambda^2$$

→ G has cutoff (or momentum) dependence

Renormalization of G_N : heat-kernel regularization

Integrate out in $g_{\mu\nu}$ -background with generally covariant regulator:

$$e^{-S_{\text{eff}}(g_{\mu\nu})} = \int \mathcal{D}\phi \, e^{-\int d^4x \, \phi(-\square_g + m^2)\phi} = [\det(-\square_g + m^2)]^{-\frac{1}{2}} \Rightarrow$$

$$S_{\text{eff}}(\mu) = \frac{1}{2} \ln \det(-\square_g + m^2) = \frac{1}{2} \sum_i \ln \lambda_i = -\frac{1}{2} \int_{\Lambda^{-2}}^{\mu^{-2}} \frac{d\tau}{\tau} H(\tau)$$

with the heat kernel $H(\tau) \equiv \text{Tr} \, e^{-\tau(-\square_g + m^2)} = \int d^4x \, G(x, x, \tau)$,
where the Green's function G satisfies:

$$\left(\frac{\partial}{\partial \tau} - \square_g^{(x)} \right) G(x, x', \tau) = 0; \quad G(x, x', 0) = \delta^{(4)}(x - x').$$

In flat space ($g_{\mu\nu} = \eta_{\mu\nu}$):

$$G_0(x, x', \tau) = \frac{1}{(4\pi\tau)^2} e^{-(x-x')^2/4\tau} \Rightarrow H_0(\tau) = \frac{1}{(4\pi\tau)^2} \int d^4x.$$

→ contribution to vacuum energy $S_{\text{eff}} \sim \int d^4x (\Lambda^4 - \mu^4)$

Renormalization of G_N : heat-kernel regularization

But in curved space background:

$$H(\tau) = \frac{1}{(4\pi\tau)^2} \left(\int d^4x \sqrt{-g} + \frac{\tau}{6} \int d^4x \sqrt{-g} R + \mathcal{O}(\tau^{3/2}) \right)$$

→ contribution $S_{\text{eff}}(\mu) \sim -\frac{1}{16\pi} \frac{\Lambda^2 - \mu^2}{12\pi} \int d^4x \sqrt{-g} R$ to

$$S_{\text{bare}} \sim -\frac{1}{16\pi} \frac{1}{G_{\text{bare}}} \int d^4x \sqrt{-g} R$$

→ Wilsonian running relation between $G(\mu)$ and $G(\mu_0)$:

$$\frac{1}{G(\mu)} = \frac{1}{G(\mu_0)} - \frac{\mu^2 - \mu_0^2}{12\pi}$$

(see also F. Larsen and F. Wilczek, Nucl. Phys. B **458**, 249 (1996))

Running of Newton's constant

Integrate out scalars, fermions and gauge bosons:

$$\frac{1}{G(\mu)} = \frac{1}{G(\mu_0)} - \frac{\mu^2 - \mu_0^2}{12\pi} (n_0 + n_{1/2} - 4n_1)$$

n_0 — number of real scalars

$n_{1/2}$ — number of Weyl fermions

n_1 — number of gauge bosons

If $N \equiv (n_0 + n_{1/2} - 4n_1) > 0$, then $G(\mu) > G_N = G(0)$.

→ Gravity becomes stronger at higher energies/shorter distances.

The *true* Planck scale

Planck scale = scale where quantum gravity effects become important

→ Planck scale μ_* : $\mu_* = G(\mu_*)^{-1/2}$,
i.e. fluctuations in spacetime geometry at
length scales $< \mu_*^{-1}$ are unsuppressed (since $\mu_* = M_{\text{Pl}}(\mu_*)$)

This μ_* is the *true* Planck scale,
"our" $M_{\text{Pl}} = G(0)^{-1/2} = 10^{19} \text{ GeV}$ derived through running effects

With $\frac{1}{G(\mu_*)} = \frac{1}{G(0)} - \frac{\mu_*^2}{12\pi} N$:

$$\mu_* = \frac{10^{19} \text{ GeV}}{\sqrt{1 + N/12\pi}}$$

$$(N = n_0 + n_{1/2} - 4n_1)$$